8.2- Arithmetic Sequences and Partial Sums

linear

geometric

exponential

A sequence is arithmetic if the differences between consecutive terms are the same.

The sequence $a_1, a_2, a_3, ..., a_n, ...$ is arithmetic if there is a number d such that

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... = d$$

The number d is the common difference of the arithmetic sum.

Determine if the sequence is arithmetic. If it is, find the common difference.

a) 7, 11, 15, 19, . . .

$d = 4$

b) 5, 12, 15, 22, 25,..

7 3 7 3

nope

c) 6.5, 5.5, 4.9, 4.3, 3.7, ...

-1 -.6

nope

The nth term of an Arithmetic Sequence

The *n*th term of an arithmetic sequnce has the form

$$a_n = dn + c$$ simplify

$$* \ a_n = a_1 + (n-1)d$$

*explicit

where *d* is the common difference between consecutive terms of the sequence and

$$c = a_1 - d$$

*recursive $a_n = a_{n-1} + d$

Find a formula for a_n for the arithmetic sequence.

a) $a_1 = 100$, $d = -8$

$a_n = 100 + (n-1)(-8)$

$a_n = -8n + 108$

recursive

$a_{n+1} = a_n - 8$

b) $a_1 = -10$, $d = -12$

c) $a_5 = 15$, $a_{12} = 36$

3 _ _ _ _ 15 _ _ _ _ _ _ 36

$36 = 15 + 7d$

$d = 3$

$\frac{15-36}{5-12} = 3$

$15 = a_1 + 4(3)$

Write the first five terms. Find the common difference and write the *n*th term as a function of *n*.

a) $a_1 = 13$, $\underline{a_{k+1} = a_k + 4}$

13, 17, 21, 25, 29

+4

Find the indicated missing term.

$a_1 = 5$, $a_2 = 11$, a_{15}

$$a_{15} = \underset{a_1}{5} + \underset{(n-1)}{(14)}\underset{d}{6}$$

The Sum of an Arithmetic Sequence

The sum of an arithmetic sequence is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

The sum of the first *n* terms of an infinite sequence is called the *n*th partial sum.

Find the sum.

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$

$S = \frac{10}{2}(1+19)$

$5(20) = 100$

$1 + 2 + 3 + 4 + 5 + 6 + \ldots + 99 + 100$

$\frac{100}{2}(1 + 100) = 50(101) = 5{,}050$

Find the partial sum.

8, 20, 32, 44, ... n=10

$a_n = a_1 + (n-1)d$

$\frac{10}{2}(8 + 116) = 620$

$a_{10} = 8 + (9)(12)$
$= 116$

7, 21, 35, 49, ... n=15

$\frac{15}{2}(7 + \quad)$

$a_{15} =$

Find the partial sum.

$$\sum_{n=1}^{100} 5n$$

$$\frac{100}{2}(5 + 500) = 50(505)$$

$$25,250$$

$$\sum_{n=1}^{500} (n+8)$$

$$\frac{500}{2}(9 + 508) = 250(517)$$

$$\sum_{n=8}^{20} n - \sum_{n=1}^{7} n$$

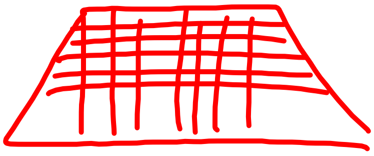
$\frac{13}{2}(8+20)$ $\frac{7}{2}(1+7)$

$182 - 28 = 154$

$$\sum_{n=1}^{15} (100-2n)$$

$\frac{15}{2}(98+70) = 1260$

A brick patio has the approximate shape of a trapezoid. The patio has 18 rows of bricks. The first row has 14 bricks and the 18th row has 31. How many bricks are in the patio?



$$\frac{18}{2}(14+31)$$

$$9(45)$$

$$405$$